IMP 3 September & October: Fireworks

Topics to be covered

Factoring and related ideas:

- Factoring quadratic expressions and using factoring to solve quadratic equations
- Identifying certain quadratic expressions as perfect squares
- Relating the multiplication of linear expressions and the factoring of quadratic expressions to problems involving area
- Understanding the conventional meaning of not factorable
- Using the zero property of multiplication in solving equations

Graphs of quadratic functions:

- Recognizing the significance of the sign of the x2-term in determining the orientation of the graph of a quadratic function
- Seeing the value of vertex form in finding the vertex of a quadratic function
- Transforming quadratic expressions into vertex form
- Understanding the role of the vertex and the x-intercept in the graphs of quadratic functions
- Using graphs to solve problems involving quadratic functions

Representing real-world problems by algebra:

- Expressing problem situations in terms of functions and equations
- Learning terminology related to quadratic expressions, functions, and equations
- Using formulas in solving problems

Philadelphia Curriculum Framework Third Edition Citations

Number Systems (IM 2) C; Patterns, Algebra & Functions (IM 2) B, C, D, E; Number Systems (IM 3) B; Geometry (IM 3) E; Patterns, Algebra & Functions (IM 3) A, D, E; Using Data, Statistics & Probability (IM 3) E; Problem Solving & Reasoning (IM 3) A, B, C, D; Communication (IM 3) A, B, C, D; Use of Tools & Technology (IM 3) A

Pa Standards Citations

PA STANDARDS: 2.2A8, 2.2B1, 2.2F, 2.4A3, 2.5A1, 2.5A2, 2.5A3, 2.5A4, 2.5B1, 2.5B2, 2.5B3, 2.5B4, 2.5C1, 2.5C2, 2.5C3, 2.5C4, 2.8D4, 2.8N2, 2.8R, 2.8S, 2.11A1, 2.11A3, 2.11A4, 2.11B

IMP 3 November & December: Orchard Hideout

Topics to be covered

Algebra:

- Using algebra in a variety of proofs involving coordinates and angles
- Using the technique of completing the square to transform equations of circles into standard form

Circles:

- Defining π and seeing why it appears in the formulas for both the circumference and area of a circle
- Developing and applying the formulas for the circumference and area of a circle
- Finding formulas for the perimeter and area for regular polygons circumscribed about a circle
- Using circumscribed polygons to see that the -circumference coefficient" for the circle is twice the "area coefficient" for the circle
- Using similarity to see that the circumference of a circle should be a constant times its radius and that the area of a circle should be a constant times the square of its radius

Coordinate geometry:

- Developing and applying the distance formula
- Developing and applying the midpoint formula
- Developing the standard form for the equation of a circle with a given center and radius
- Finding the distance from a point to a line in a coordinate setting
- Using the Cartesian coordinate system to organize a complex problem

Logic:

- Understanding and using the phrases "If .... then ... " and "if and only if" in definitions and proofs
- Working with converses

Miscellaneous:

- Learning about Pythagorean triples
- Using symmetry to help analyze a problem

Synthetic geometry:

- Defining the distance from a point to a line and proving that the perpendicular distance is the shortest
- Discovering and proving that any line through the midpoint of a segment is equidistant from the endpoints of the segment
- Discovering and proving that the set of points equidistant from two given points is the perpendicular bisector of the segment connecting the given points
- Discovering and proving that the set of points equidistant from two intersecting lines is the union of the bisectors of the angles formed by the lines
- Identifying and describing a set of points satisfying a geometric condition

PA STANDARDS: 2.2A8, 2.2B1, 2.2F, 2.4A2, 2.4A3, 2.4E5, 2.5A1, 2.5A2, 2.5A3, 2.5A4, 2.5B1, 2.5B2, 2.5B3, 2.5B4, 2.5C1, 2.5C2, 2.5C3, 2.5C4, 2.8A3, 2.8D4, 2.8E1, 2.8E2, 2.8J1, 2.8J3, 2.8J4, 2.8R, 2.9B1, 2.9B2, 2.9D1, 2.9E1, 2.9E2, 2.9E3, 2.9E4, 2.9F1, 2.9F2, 2.9F3, 2.9F6, 2.9G1, 2.9G2, 2.9G3, 2.9G5, 2.9I

IMP 3 January & February: Meadows or Malls?

Topics to be covered

General linear programming:

- Generalizing the corner-point principle to more than two variables
- Generalizing the method of finding corner points to more than two variables
- Recognizing that for two-variable problems, corner points can be found as the intersections of lines corresponding to constraint equations or inequalities
- Seeing that for two-variable problems, the optimal value always occurs at a corner point of the feasible region

Geometry in the plane and in 3-space:

- Examining the possible intersections of planes in 3-space
- Relating the possible intersections of lines and planes to the algebra of solving linear systems in two or three variables
- Seeing that two distinct points always determine a unique line and that two distinct lines in the plane determine a unique point unless the lines are parallel

Matrices and systems of linear equations:

- Finding matrix inverses by hand by solving systems of linear equations
- Recognizing the role of identity and inverse elements in solving certain types of matrix equations
- Seeing that systems of linear equations are equivalent to certain types of matrix equations
- Understanding the relationship between a system of linear equations having a unique solution and the coefficient matrix being invertible

Matrix algebra:

- Examining whether matrix operations have certain properties such as associativity and commutativity
- Using matrices to represent information
- Using problem situations to motivate and develop the definitions of matrix addition and multiplication

Solving linear equations:

- Using substitution, graphing, and guess-and-check methods to solve systems of linear equations in two variables
- Developing and using the elimination method to solve systems of linear equations in two or more variables
- Using the concepts of inconsistent, dependent, and independent systems of equations

Technology:

- Entering matrices and doing matrix operations on a graphing calculator
- Using matrix inversion on a graphing calculator to solve systems of linear equations

Three-variable coordinate system:

- Extending the concept of coordinates to three variables by introducing a third axis perpendicular to the first two
- Graphing linear equations in three variables and recognizing that these graphs are planes in 3-space

PA STANDARDS: 2.1A7, 2.2A8, 2.2B1, 2.2F, 2.4A3, 2.5A1, 2.5A2, 2.5A3, 2.5A4, 2.5B1, 2.5B2, 2.5B3, 2.5B4, 2.5C1, 2.5C2, 2.5C3, 2.5C4, 2.8D1, 2.8D2, 2.8D3, 2.8D4, 2.8F1, 2.8F2, 2.8G1, 2.8G2, 2.8H1, 2.8H2, 2.8H3, 2.8H4, 2.8I1, 2.8I2, 2.8K1, 2.8K2, 2.8K5, 2.8K6, 2.8R

IMP 3 March & April: Small World, Isn't It?

Topics to be covered

Derivatives:

- Developing the concept of the derivative of a function at a point
- Finding numerical estimates for the derivatives of functions at specific points
- Realizing that for functions of the form y = bx, the derivative at each point of the graph is proportional to the y-value at that point
- Seeing that the derivative of a function at a point is the slope of the tangent line at that point
- Working with the derivative of a function as a function in itself

Exponential and logarithmic functions:

- Discovering that any exponential function can be expressed using any positive number other than 1 as a base
- Learning the meaning of the terms natural logarithm and common logarithm
- Reviewing and applying the principles that $a^b \bullet a^c = a^{b+c}$ and $(a^b)^c = a^{bc}$
- Strengthening understanding of logarithms
- Understanding and using the fact that $_a{}^{(\log_a b)} = b$
- Using an exponential function to fit a curve to numerical data
- Using exponential functions to model real-life situations

Rate of change:

- Evaluating average rate of change in terms of the coordinates of points on a graph
- Realizing that, other things being equal, the rate of change in population is proportional to the population
- Understanding the relationship between the rate of change of a function and the appearance of its graph

Slope and linear functions:

- Developing an algebraic definition of slope
- Proving, using similarity, that lines have it constant slope
- Seeing that the slope of a line is equal to the coefficient of x in the algebraic representation of the line
- Understanding the significance of a negative slope in terms of the appearance of a graph
- Using slope to develop equations for straight lines

The number *e* and compound interest

- Developing and using it formula for Compound interest
- Estimating the value of b for which the function, y = bx has a derivative at each point on its graph equal to the y-value at that point
- Learning that the limiting value *e* is the same number as the special base for exponential functions
- Seeing that expressions of the form $(1 + 1/n)^n$ have a limiting value, called *e*

PA STANDARDS: 2.1A12, 2.2A8, 2.2B1, 2.2F, 2.3A4, 2.4A3, 2.4E6, 2.5A1, 2.5A2, 2.5A3, 2.5A4, 2.5B1, 2.5B2, 2.5B3, 2.5B4, 2.5C1, 2.5C2, 2.5C3, 2.5C4, 2.8D4, 2.8K1, 2.8K2, 2.8K4, 2.8L1, 2.8L2, 2.8N4, 2.8N7, 2.8R, 2.8S, 2.8T1, 2.11C1, 2.11C2, 2.11C3, 2.11C4, 2.11D1

IMP 3 May & June: Pennant Fever

Topics to be covered

Counting principles:

- Defining and using the concepts of permutation and combination
- Developing formulas for the permutation and combinatorial coefficients
- Developing systematic lists for complex situations
- Understanding and using standard notation for counting permutations and combinations
- Using the multiplication principle for choosing one element from each of several sets

Pascal's triangle and combinatorial coefficients:

- Finding patterns and properties within Pascal's triangle
- Recognizing that Pascal's triangle consists of combinatorial coefficients
- Developing and explaining the binomial theorem
- Explaining the defining pattern and other properties of Pascal's triangle using the meaning of combinatorial coefficients

Probability and statistics:

- Developing a mathematical model for a complex probability situation
- Finding and using probabilities for sequences of events
- Finding expected value
- Using a simulation to understand a situation, to help analyze probabilities, and to support a theoretical analysis
- Using area diagrams and tree diagrams to find and explain probabilities
- Using probability to evaluate null hypotheses
- Using specific problem contexts to develop the binomial distribution, and finding a formula for the associated probabilities

PA STANDARDS: 2.1A8, 2.2A3, 2.2A8, 2.2A9, 2.2A11, 2.2A12, 2.2B1, 2.2B4, 2.2B6, 2.2F, 2.4A3, 2.5A1, 2.5A2, 2.5A3, 2.5A4, 2.5B1, 2.5B2, 2.5B3, 2.5B4, 2.5C1, 2.5C2, 2.5C3, 2.5C4, 2.7E3, 2.7E4, 2.7E5, 2.8D4, 2.8R